Summary and Outlook: Clarifications, Elucidations & Future directions

• Quantum gravity phenomenology: A success story. Useful results seemed out of reach ten years ago but we now have interesting and surprisingly strong bounds. Analogy with Will's classification of Relativistic Gravitational Theories with PPN parameters in the seventies. Powerful techniques to weed out otherwise viable theoretical ideas. Rich interplay between observations and hard theory.

• Euclidean Quantum Gravity: Non Gaussian Fixed point
$\star$ Issue of units *Peculiarity of GR: field rescaling and momentum rescaling are the same operations* $\Rightarrow$ Can use Planck units (G fixed, does not run) or cut-off units (cut-off brought to $k = 1$ at each step; G runs). Planck units used in non-perturbative approaches. But in these units β functions depend on k. What is then the notion of a fixed point in the Planck units?
$\star$ *Clarifications:* Scale dependent metric; Physical understanding of scheme dependence.
$\star$ *Possible connections with LQG and Dynamical triangulations:* Effective dimension?; Running of G in LQC ($\rho = \rho_{\rm crit} \leftrightarrow$ NGFP?) Lessons for Coarse graining and semi-classical sectors?

The fact that in LQG one begins with Einstein gravity in no way contradicts the higher order and even non-local corrections.

• Effective field theory: Integrating gravitational DOF
Connection with: Work of the Madrid group? More models? NCG seems closely tied to 3-d structure of 3-momenta (Lack of asymptotic flatness in the habitual sense). What are the relevant structures in higher dimensions?

• Spin-foams: Graviton Propagator; Many open issues but clearly the beginning of a very important development; Already new insights that could be exploited: Spaziale's improvement of the 'flat' semi-classical state.
Applications to Yang-Mills theory; a potentially rich avenue (but hard in 4-d!). In the 2-d YM action is invariant & the 'complete solution' was provided by LQG methods. Important to follow-up on the possibility of constructing confinement-arguments!

• Cosmology: More active attention to data and physical problems. Important to pursue. Need to enhance work on all three fronts: speculative ideas, hard mathematical physics results, conceptual issues. Saw examples in each category. Interface? Tricky issues when dealing with the Cosmology community.

• Singularity resolution: Clarifications
⋆ In mini and midi-superspaces, singularity resolved for *all* quantum states. In many instances, quantum bounce for states which are semi-classical at late times.
Simple but solid examples of success story —much richer than say 3-d gravity. Physical sector constructed; interesting Dirac observables; rigorous semi-classical states: issue of whether there are enough semi-classical states settled. Hard part: Getting a good UV *and* IR behavior was tricky but has been achieved. (μ_o versus $\bar{\mu}$ evolutions)
⋆ Robustness: $k = 0$, $k = 1$, Λ, certain anisotropies (interior of the Schwarzschild horizon); Pablo's talk (Adhoc Higher order modifications to dynamics do not affect the singularity resolution.)
⋆ Effective equations: Contrast with string theory tachyon condensation scenario.
⋆ The physical motivation behind the strategy of defining the non-local field operator: Incorporating the creation of new vertices in full LQG. (Discussion following Zapata's talk.)

• Dynamics: Status.

⋆ Original Thiemann procedure: Establishes that the quantization program can be completed. However do not know if the resulting theory is 'useful' i.e. has sufficiently many semi-classical states.

⋆ Ambiguities. Don't go away in the Master constraint program. But once a choice of the constraint operator is made, the program makes it easier to carry out the group averaging and show the existence of the physical inner product etc. Important limitation: don't know the *physical* difference between various choices.

⋆ in the Master Constraint Program: Commutator between original constraints? Constraint algebra?

⋆ However, these ambiguities are unrelated to those of perturbative non-renormalizability.

• Why is the progress slow?

Too ambitious programs. Goal to solve in one go issues of: quantum mechanics, quantum gravity, improving the uv behavior of field theories; resolution of singularities. More progress likely if one sets partial but concrete landmarks. In my view, the LQG approach to quantum dynamics also suffers from the desire to solve all dynamics in one go.

• Dynamics: Current Strategies.
Another avenue: Explore the semi-classical sector in greater detail and establish correctness of the simplified Hamiltonian constraint. Difficult to articulate quickly but I think there is a convergence of ideas of Bojowald (inhomogeneities in cosmology), Gambini-Pullin (uniform discretization), Theimann (Graph preserving regularization) and mini and midi superspaces. For example: Could gauge fix the diff constraint (eg 'the metric be diagonal') and then look at the resulting Hamiltonian constraint alone. Will not be global gauge fixing but should work in large patches of phase space (strategy of geometric analysis in classical GR to study approach to singularity; Moncrief, Rendall, Garfinkle,...).

• Other Fertile Directions:
BH evaporation
Midi-superspaces: Gowdy Universe
Numerical Quantum Cosmology
Singularity Resolution Theorems.
Matter from braids??

Work on numerous fronts essential for overall progress: Mathematically precise and exact results, approximations and truncations and speculative ideas. However, important to maintain clarity on the status of results both in research *and* presentations/papers.