Randall-Sundrum with Kalb-Ramond field: return of the hierarchy problem?

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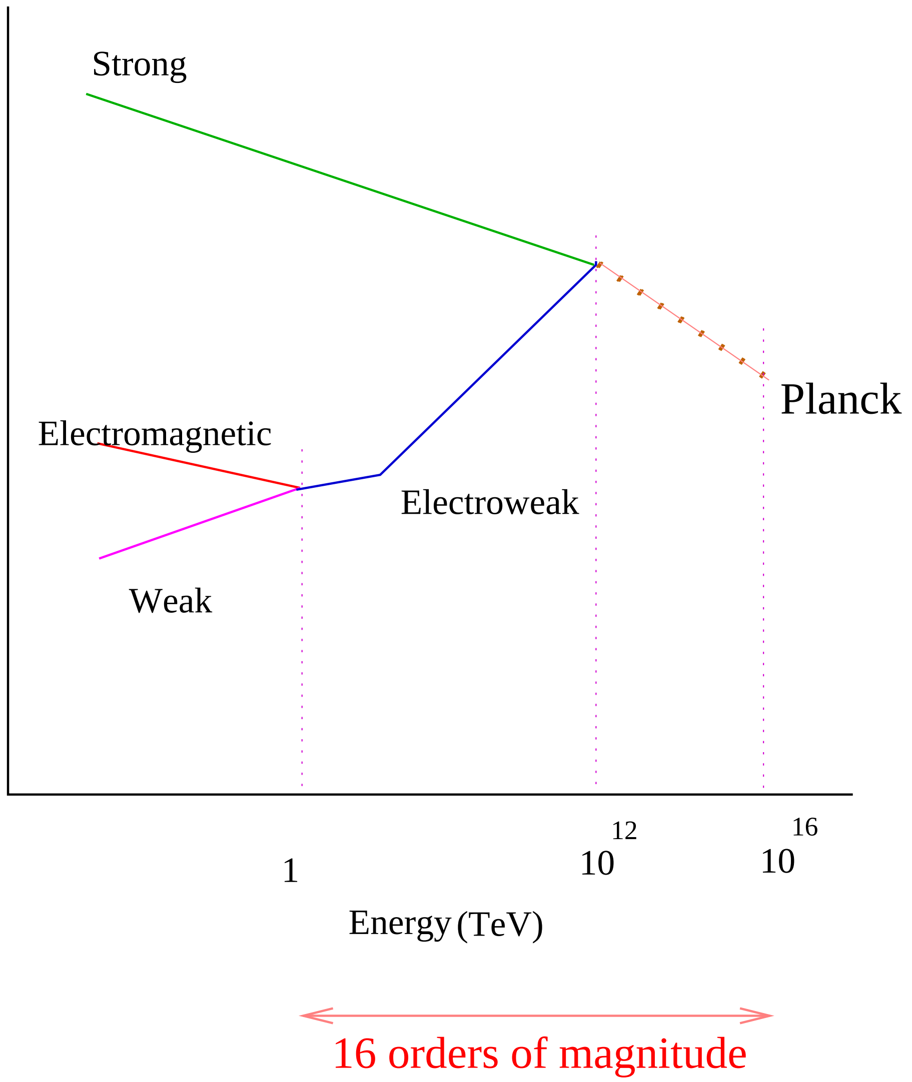
CANADA

PLAN:

- The hierarchy problem and RS brane world scenario
- The Kalb-Ramond field and the return of the hierarchy problem
- Induced negative 4 dimensional cosmological constant

Reference:

S. Das, A. Dey, S. SenGupta, hep-th/0511257, to appear in Class. Quant. Grav. (Letters)


Strong
Planck
Electromagnetic
Electroweak
Weak
1
10
12
10
16
Energy (TeV)
16 orders of magnitude

TeV Planck scale from Large Extra Dimensions

No of Spacetime dimensions $$d = \underbrace{4}_{Observed} + \underbrace{(d-4)}_{Unobserved}$$

Standard Model & Gravity — Gravity

RS Scenario:

Einstein action in d dimensions:

$$S = \frac{c^3}{16\pi G_d} \int d^d x \sqrt{-g_d}\ \mathcal{R}_d$$

Assume: $ds_d^2 = e^{-A(y)} \underbrace{ds_4^2}_{Observed} - \underbrace{dy_I dy^I}_{Unobserved}$

↑ non-trivial role of y_Is:

Then:

$$S = \frac{\left[c^3 \int d^{d-4}y \sqrt{g(y)} e^{-A}\right]}{16\pi G_d} \int d^4 x \sqrt{-g_d}\ \mathcal{R}_4 = \frac{c^3}{16\pi G_4} \int d^4 x \sqrt{-g_4}\ \mathcal{R}_4$$

$$\boxed{G_4 = G_d \left[\int d^{d-4}y \sqrt{g(y)} e^{-A}\right]^{-1}}$$

$$\boxed{M_{Pl(d)}^{d-2} = \left(\frac{\hbar}{c}\right)^{d-4} \frac{M_{Pl(4)}^2}{\int d^{d-4}y \sqrt{g(y)} e^{-A}} \approx \left(\frac{\hbar}{c}\right)^{d-4}\ k^{d-4}\ M_{Pl(4)}^2}$$

$[A = kL]$

However:

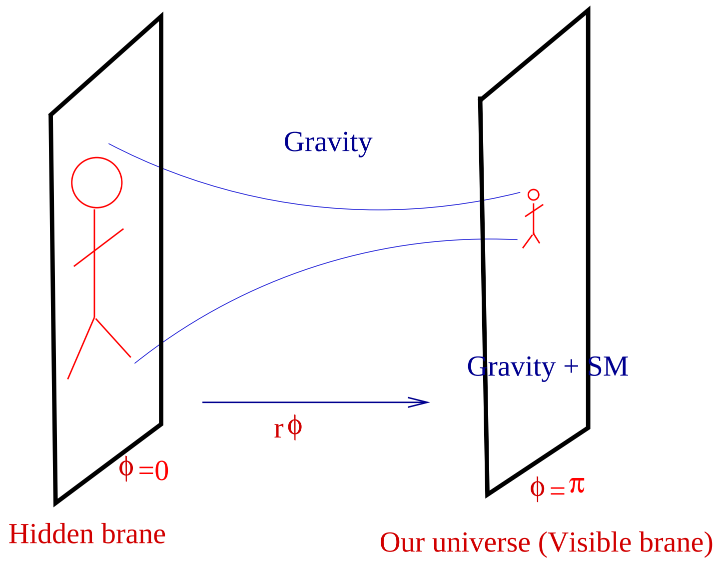
If our universe ('brane') is located at $y = y_0$, metric has conformal factor $\Omega^2 = e^{-A(y_0)}$

Physical masses $M_{(4)}^{phys} = e^{-A(y_0)} M_{(4)}^{param}$

Thus if $M_{(4)}^{param}\ c^2 = 10^{16} TeV$ and $A \approx 12$

Then:
$M_{(4)}^{phys} c^2 = 1\ TeV$

A Small conformal factor is sufficient to solve hierarchy



Computing the warp factor

Ansatz:

$$ds^2 = e^{-A}\ \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2 \ \leftarrow \ \text{extra dim}$$

Action ($M_{Pl(5)} \equiv M, \mathcal{R}_5 = R$):

$$S \ = \ S_{Gravity} + S_{vis} + S_{hid}$$

$$\text{where,} \qquad S_{Gravity} \ = \ \int d^4x \ r \ d\phi \sqrt{-G} \ [2M^3 R + \underbrace{\Lambda}_{5-d}]$$

$$S_{vis} \ = \ \int d^4x \sqrt{-g_{vis}} \ [L_{vis} - V_{vis}]$$

$$S_{hid} \ = \ \int d^4x \sqrt{-g_{hid}} \ [L_{hid} - V_{hid}]$$

Eq. of motion:

$$\frac{3}{2} A'^2 \ = \ -\frac{\Lambda}{4M^3}\ r^2 \qquad \left[' = \frac{d}{d\phi}\right]$$

Solution:

$$A \ = \ 2kr\phi$$

$$V_{hid} \ = \ -V_{vis} = 24M^3 k \qquad \left[k = \frac{-\Lambda}{24M^3}\right]$$

Warping

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}\big|_{\phi=\pi} = e^{-2kr\pi} \approx (10^{-16})^2$$

$$\Rightarrow kr = \frac{16}{\pi}\ln(10) = 11.6279\ldots \quad \leftarrow \text{RS value}$$

What if there are other fields in the bulk?

Branes in String Theory ? $\Rightarrow$ D-branes?

Massless modes in string theory

$$\alpha^{\mu}_{-1}\ \tilde{\alpha}^{\nu}_{-1}|0;k\rangle \quad \leftarrow \text{Massless}$$

$\downarrow$	$\downarrow$	$\downarrow$
Symmetric	Anti-symmetric	Trace
$\downarrow$	$\downarrow$	$\downarrow$
Graviton $g_{\mu\nu}$	Kalb-Ramond $B_{\mu\nu}$	Dilaton ϕ
$\downarrow$	$\downarrow$	$\downarrow$
Already present in bulk	???	???
	$\uparrow$	
	Include here	

<u>Note:</u>

$$H_{MNL} = \partial_{[M}B_{NL]}$$

Computing the warp factor with bulk KR field

Ansatz:

$$ds^2 = e^{-A}\ \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2 \ \leftarrow \ \text{extra dim}$$

Action:

$$S \ = \ S_{Gravity} + S_{vis} + S_{hid} + S_{KR}$$

$$\text{where,} \quad S_{Gravity} \ = \ \int d^4x \ r \ d\phi \sqrt{-G} \ [2M^3 R + \underbrace{\Lambda}_{5-d}]$$

$$S_{vis} \ = \ \int d^4x \sqrt{-g_{vis}} \ [L_{vis} - V_{vis}]$$

$$S_{hid} \ = \ \int d^4x \sqrt{-g_{hid}} \ [L_{hid} - V_{hid}]$$

$$S_{KR} \ = \ \int d^4x \ r \ d\phi \sqrt{-G} \ [-2H_{MNL}H^{MNL}] \leftarrow \text{Additional}$$

Eq. of motion:

$$\frac{3}{2}A'^2 \ = \ -\frac{\Lambda}{4M^3}\ r^2 - \frac{3}{2M^3} g^{\nu\beta} g^{\lambda\gamma} H_{\phi\nu\lambda} H_{\phi\beta\gamma} \ \leftarrow \ \underline{\text{New}}$$

$$\frac{3}{2}(A'^2 - A'') \ = \ -\frac{\Lambda}{4M^3} r^2 + \frac{\exp(-2A)}{2M^3}\ \eta^{\lambda\gamma}[-12\eta^{00} H_{\phi 0\lambda} H_{\phi 0\gamma} + 3\eta^{\nu\beta} H_{\phi\nu\lambda} H_{\phi\beta\gamma}]$$

$$\frac{3}{2}(A'^2 - A'') \ = \ -\frac{\Lambda}{4M^3} r^2 + \frac{\exp(-2A)}{2M^3}\ \eta^{\lambda\gamma}[-12\eta^{ii} H_{\phi i\lambda} H_{\phi i\gamma} + 3\eta^{\nu\beta} H_{\phi\nu\lambda} H_{\phi\beta\gamma}]$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

$$\underline{\text{New}} \qquad\qquad \underline{\text{New}}$$

Solution:

$$e^{-A} = \frac{\sqrt{b}}{2kr} \cosh\left(2kr\phi + 2krc\right)$$

$b \sim$ Energy density of KR field

$$\frac{2kr}{\sqrt{b}} = \cosh(2krc) \quad , \quad \text{such that } A(0) = 1$$

$$c = -\frac{1}{2kr} \tanh^{-1}\left(\frac{V_{hid}}{24M^3k}\right) = -\pi + \frac{1}{2kr} \tanh^{-1}\left(\frac{V_{vis}}{24M^3k}\right)$$

RS Limit:

$b \rightarrow 0$, $c \rightarrow -\infty$

Warping

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}|_{\phi=\pi} = \frac{\sqrt{b}}{2kr}\cosh\left[2kr\pi + \cosh^{-1}\frac{2kr}{\sqrt{b}}\right] \quad \leftarrow \text{New warp factor}$$

$$= \left[\cosh(2kr\pi) - \sinh(2kr\pi)\sqrt{1-\frac{b}{(2kr)^2}}\,\right]$$

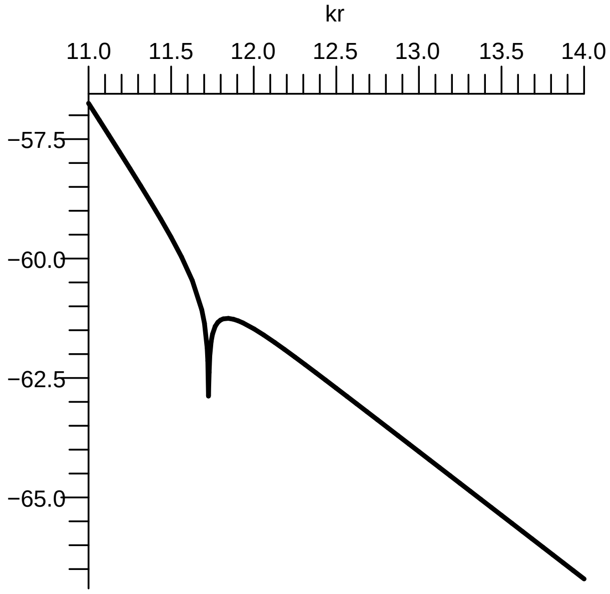
$$\approx (10^{-16})^2$$

Invert to get $b = b(kr)$:

$$b = (2kr)^2\left[1 - \left(\coth(2kr\pi) - (m_H/m_0)^2\mathrm{cosech}(2kr\pi)\right)^2\right]$$

$kr =$ RS value $\Rightarrow$ $b = 0$
$kr >$ RS value $\Rightarrow$ $b > 0$
$kr <$ RS value $\Rightarrow$ $b < 0$ $\leftarrow$ Unphysical, since then $\left(\frac{m_H}{m_0}\right)^2 \in \mathcal{C}$

Therefore, $b > 0$, but how large ?



$\log|b|$ vs kr, for $\frac{m_H}{m_0} = 10^{-16}$

kink = RS value of kr ($b = 0$)

Left of kink: $kr <$ RS value, $b < 0$ (unphysical)

Right of kink: $kr >$ RS value, $b > 0$ (physical)

Any value of $kr >$ RS value solves hierarchy

<u>But</u>

$\boxed{b_{max} = 10^{-62}}$

$\uparrow$

Extreme fine tuning of b !!

Induced 4-dim cosmological constant (0 for RS)

$$^{(5)}G_{AB} = -\Lambda \; ^{(5)}g_{AB} + 8\pi G_5 \; T_{AB}$$

$$\Downarrow$$

$$G_{\mu\nu} = -\lambda \; g_{\mu\nu} + \ldots$$

$$\lambda \equiv \frac{1}{2}\left(kV_{vis} + \Lambda\right) \; = \; -12M^3k\left[\tanh(2krc) + 1\right]$$

$$\approx \; -24M^3k \; \frac{b}{(4kr)^2} \;\; \approx -10^{-63} \neq +10^{-123} \leftarrow \text{ accepted value}$$

SUMMARY AND OUTLOOK:

- **RS model solves hierarchy problem with gravity and KR field in the bulk**
- *BUT* **Extreme fine tuning of KR field:** 1 **part in** $10^{62} \leftarrow$ *Return of the hiearchy/fine-tuning problem*
- **Wrong sign of induced** 4**-dim cosmological constant**
- **Include dilaton, other fields?**
- **Is RS brane world the answer??**