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# "Non-Equilibrium Thermodynamics of Spacetime"

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- What are the states that bh entropy counts?
- ~~[illegible]~~ Bh entropy or horizon entropy?
- Newton's constant and bh entropy: renormalization?
- ~~is gravity thermodynamical~~, ~~different~~?

~~[illegible]~~

10 yrs old derivation of EE as eq'n of state based on some hypotheses:

(1) all local causal horizons have a universal entropy density $\alpha$

$$S = \alpha A.$$

(2) Clausius rel'n:

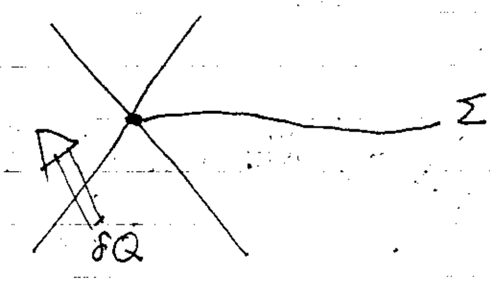
$$dS = \frac{\delta Q}{T}$$

$\delta Q$ = boost energy flux across horizon.

$T = \frac{\hbar}{2\pi}$ : Unruh temperature ~~of horizon~~

$$\Rightarrow \quad R_{ab} - \tfrac{1}{2} R g_{ab} - \Lambda g_{ab} = 8\pi G T_{ab}.$$

$\Lambda$ arbitrary, $G = (4\hbar\alpha)^{-1}$




Focusing of light rays $\rightarrow$ gravity

shearing

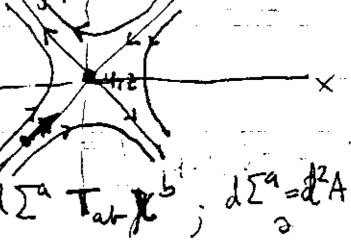
Explain it here plus recent extension done w/ Chris Eling and Raf Guedens
- incorporate non-equilibrium viscosity terms. shear.
- incorporate ~~higher~~ curvature corrections to entropy (what led us to non-equil).

on horizon: Minkowski spacetime, boost symmetry

$K^a = \left(\frac{\partial}{\partial \lambda}\right)^a$; if $\lambda$ = aff parameter, 0 at $p$, $\chi = x\,\partial_t + t\,\partial_x$ ; bifurcation plane $x = t = 0$.

$v$ = Killing parameter

$\Rightarrow \chi^a = -\lambda K^a.$




$\chi^a = \left(\frac{\partial}{\partial v}\right)^a$

$\lambda = -e^{-v}$

$\frac{d\lambda}{dv} = -\lambda$

$$\mathrm{Tr}_L |0\rangle\langle 0| = Z^{-1} \exp\left(-\frac{2\pi}{\hbar} H_\chi\right)$$

$$H_\chi = \int d\Sigma^a \, T_{ab} \chi^b \; ; \; d\Sigma^a = d^2A\, k^a d\lambda$$

Unruh effect: the root of bh thermo?

(2)

Curved spacetime: "local boost symmetry"

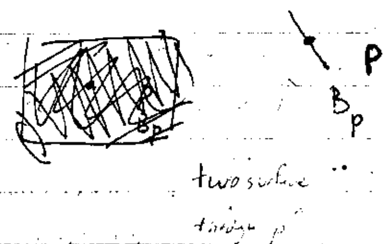
Killing's eq'n $\chi_{a;b} + \chi_{b;a} = 0$.

Sol'n determined by $\chi_a$, $\chi_{a;b}$ at one point.

Minkowski boost: $\chi_a\big|_p = 0$ $\quad \chi_{a;b}\big|_p = \begin{array}{c|cccc} & t & x & y & z \\ \hline t & 0 & +1 & 0 & 0 \\ x & -1 & 0 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$

$\chi_{a;b} \perp$ Bifurcation plane.

$\chi_{a;b}\chi^{a;b} = -2$.




try to solve Killing's eq'n in Riemann normal coord's –

$\Rightarrow$ $2^{nd}$ order terms vanish

$3^{rd}$ " " cannot be satisfied

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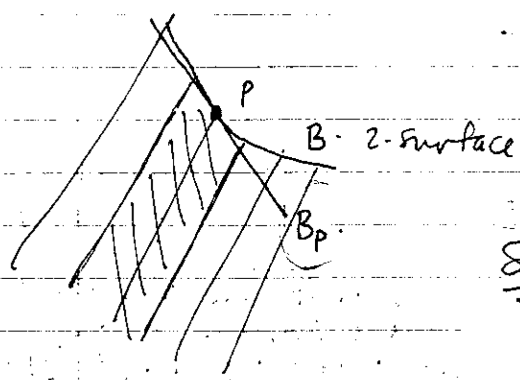


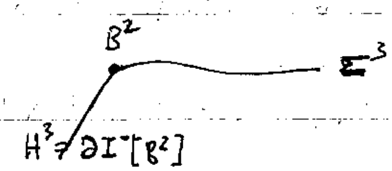


$$\frac{\delta Q}{T} = \int T_{ab}\underbrace{\chi^a}_{O(\lambda)} d\Sigma^b$$

$$= \frac{c_1}{t}\int T_{ab}k^a k^b(-\lambda)\, d\lambda\, d^2A$$

$$\delta A = \int \theta\, d\lambda\, d^2A \quad , \quad \theta = \frac{d\ln(d^2A)}{d\lambda}$$

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$

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$\theta_p = 0 \quad O(\lambda)$ part must vanish.

$$\delta S = \alpha \int \left( R_{ab} k^a k^b + \sigma_{ab}\sigma^{ab} \right) (-\lambda)\, d\lambda\, d^2A$$

$$dS = \frac{\delta Q}{T} \Rightarrow R_{ab} k^a k^b + \sigma_{ab}\sigma^{ab} = \frac{2\pi}{\hbar\alpha} T_{ab} k^a k^b \quad \forall k.$$

$$\Rightarrow \sigma_{ab} = 0 \quad \& \quad R_{ab} + \Phi g_{ab} = \frac{2\pi}{\hbar\alpha} T_{ab}.$$

previously just assumed as "equilibrium", but not so. in any case it's a trans'n to equilibrium... but how quickly?

$$\left.\begin{matrix}\text{Bianchi} \\ \& \\ T_{ab;}{}^{b} = 0\end{matrix}\right\} \Rightarrow \Phi = -\frac{1}{2} R \cancel{+} \Lambda$$

$$\frac{2\pi}{\hbar\alpha} = 8\pi G \Rightarrow \boxed{G = \frac{1}{4\hbar\alpha}.}$$

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Not as quickly $\quad \sigma_{ab}$ : shear wrt affine parameter

$\hat{\sigma}_{ab}$ : " " Killing parameter

$$= \frac{d\lambda}{dv} \sigma_{ab}$$

$$= e^{-v} \sigma_{ab}.$$

internal ~~entropy~~ production: $dS = \frac{\delta Q}{T} + d_i S$

$$\frac{d_i S}{dv\, d^2A} = \frac{1}{T} \cdot 2\zeta\, \hat{\sigma}_{ab} \hat{\sigma}^{ab}.$$ like fluid exactly.

$\zeta$: shear viscosity

so $\frac{1}{T} \cdot 2\zeta = \alpha$

$$\sigma_{ab}\sigma^{ab}(-\lambda)\, d\lambda = \hat{\sigma}_{ab} \hat{\sigma}^{ab}\, dv \qquad \zeta = \frac{\alpha T}{2} = \frac{\hbar\alpha}{4\pi} = \frac{1}{16\pi G}$$

Include bulk viscosity?? $\theta_p^2$ : NO. Why not?

Include curvature corrections to entropy.

$$S = \alpha f(R) = \alpha\left(1 + O(R)\right)$$

$$\delta S = \alpha \int \frac{d}{d\lambda}(f\, d^2A)\, d\lambda$$

$$= \alpha \int (\theta f + \dot{f})\, d\lambda\, d^2A .$$

$$dS = \frac{\delta Q}{T} \Rightarrow (\theta f + \dot{f})\Big|_p = 0 .$$

$$\Rightarrow \theta_p = -\frac{\dot{f}}{f} \neq 0 \;! \qquad (\text{Raf}).$$

Go thru similar argument & find

$$f R_{ab} - f_{;ab} + \frac{3}{2} f^{-1} f_{,a} f_{,b} + \Psi g_{ab} = \frac{2\pi}{\hbar\alpha} T_{ab} .$$

$$\Psi = \Box f - \tfrac{1}{2}\mathcal{L} - \Theta , \quad \frac{d\mathcal{L}}{dR} = f \; \&.$$

$$\left.\begin{array}{c} \text{Bianchi} \\ \& \\ T_{ab;}{}^{b} = 0 \end{array}\right\} \Rightarrow \underbrace{\left(\tfrac{3}{2} f^{-1} f_{,a} f_{,b}\right)_{;}{}^{a} = \Theta_{,b}}_{\text{NOT TRUE!}} \quad \text{some } \Theta$$

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Bulk viscosity term

$$\frac{d_i S}{d\sigma\, d^2 A} = \frac{3}{2} \alpha f \hat{\theta}^2$$

$\rightarrow$ Bulk viscosity $\eta = \frac{3}{2} \alpha f T$

$$= \frac{3}{16\pi G} f.$$

Then derive Einstein eq'n ~~in presence~~ associated w/ Lagrangian $\mathcal{L}$ that yields $S = \alpha f A \left( \propto \frac{d\mathcal{L}}{dR} \right)$.

$\frac{d\mathcal{L}}{dR}$ ((Wald entropy)).

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Comments:

① Clausius + 1st law $\rightarrow$ EOS.

$\updownarrow$

$\nabla_a T^{ab} = 0$.

② viscosity a phenomenological coefficient ?

yes, in terms of $\alpha$. e.g. shear $\zeta = \frac{\hbar \alpha}{4\pi}$.

[Derivation in string theory via AdS/CFT ?]

③ Curvature corrections meaningful ? If imagine minimum size $\epsilon$ of region for argument, Killing approximation fails ... ok if coeff of curvature term large enough ($> \epsilon^2$).

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(4) Other types of curvature terms, e.g. Lovelock?

The End