Numerics of the Big Quantum Bounce: The Shallow Waters of the Big Bang

Pablo Laguna
Penn State

GOAL: to gain further insight about the nature of the Big Quantum Bounce from a numerical analysis point of view.

LQC Finite Difference Equation

$$\partial_\phi^2 \Psi_j = \tfrac{3\kappa}{2} |v| D_v \big(|v| D_v \Psi_j \big)$$

where

$$D_v f_j \equiv \tfrac{f_{j+1/2} - f_{j-1/2}}{\Delta v}$$

v : Eigenvalue of the volume operator

in the limit $\Delta v \to 0$

$$\partial_\phi^2 \Psi = \tfrac{3\kappa}{2} v \partial_v \big(v \partial_v \Psi \big)$$ Wheeler-DeWitt Eqn

similar to Shallow Water Eqn

$$\partial_t^2 h = d\, \partial_x (g\, \partial_x h)$$

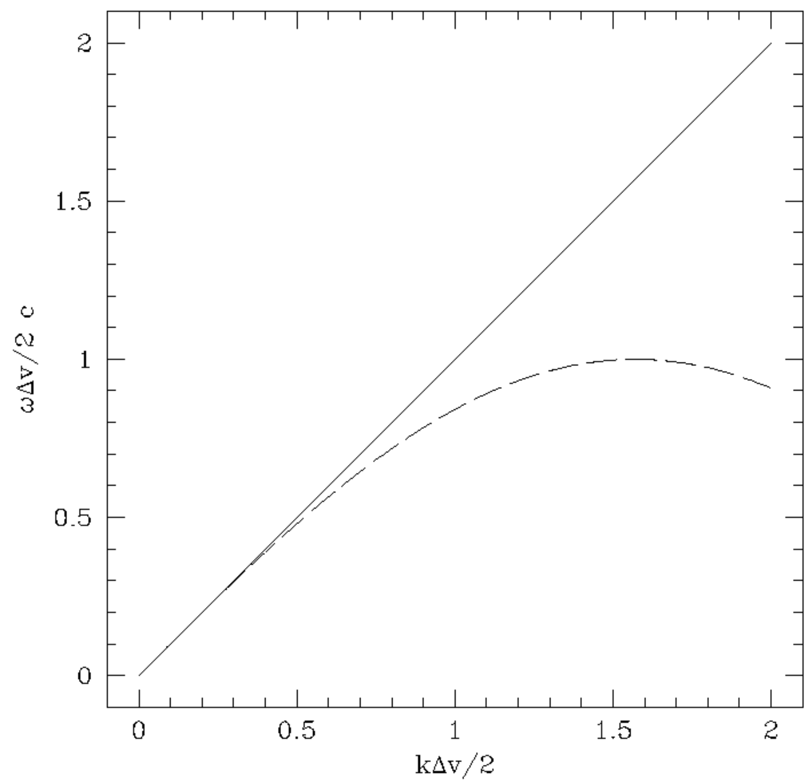
Dispersion Relation

only principal part is needed

$$\partial_\phi^2 \Psi_j = c^2 D_v^2 \Psi_j$$

$$c = \sqrt{\tfrac{3\kappa}{2}}\,|v|$$

$$\Psi_j \propto e^{i(\omega\,\phi \pm k\,j\,\Delta v)} \rightarrow \boxed{\frac{\Delta v\,\omega}{2\,c} = \pm \sin\left(\frac{\Delta v\,k}{2}\right)}$$

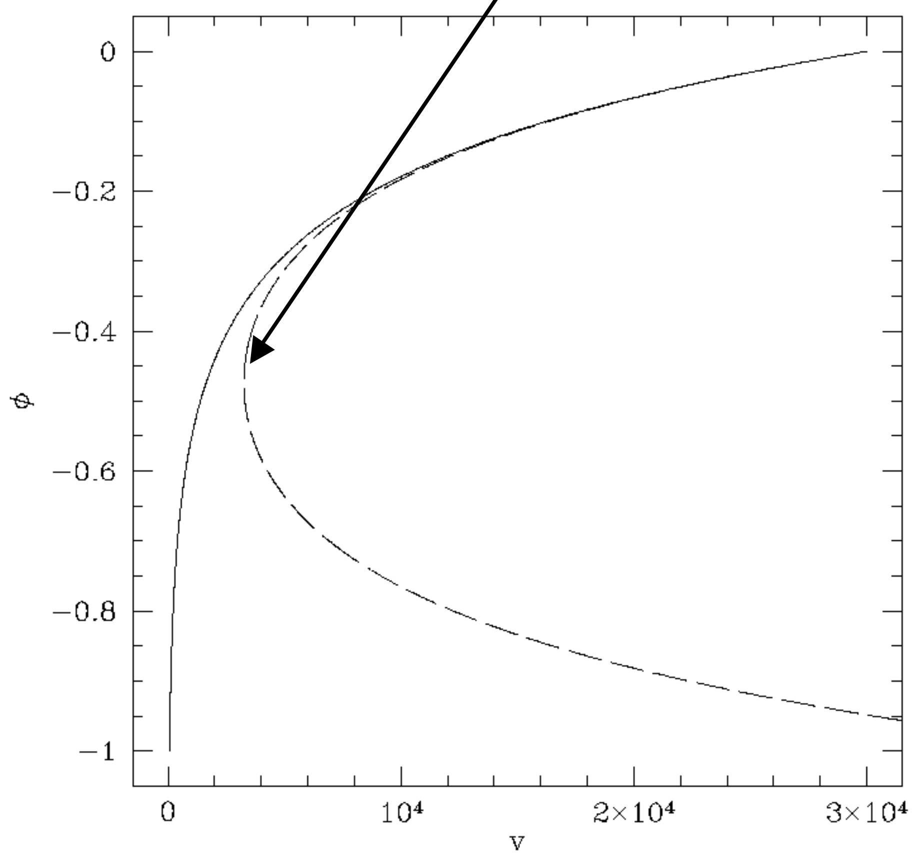


Characteristics & Group Velocity

$$V_g = \frac{d\omega}{dk} = \pm c\,\cos\left(\frac{\Delta v\,k}{2}\right) = \pm c\left[1 - \left(\frac{\Delta v\,\omega}{2\,c}\right)^2\right]^{1/2}$$

The Big Quantum Bounce

$$V_g = 0 \text{ @ } \left(\frac{\Delta v \omega}{2\,c}\right)^2 = 1$$



Phenomenology

Implicit Time-Update

$$M_v^2 \partial_\phi^2 \Psi_j = \frac{3\kappa}{2} |v| D_v (|v| D_v \Psi_j)$$

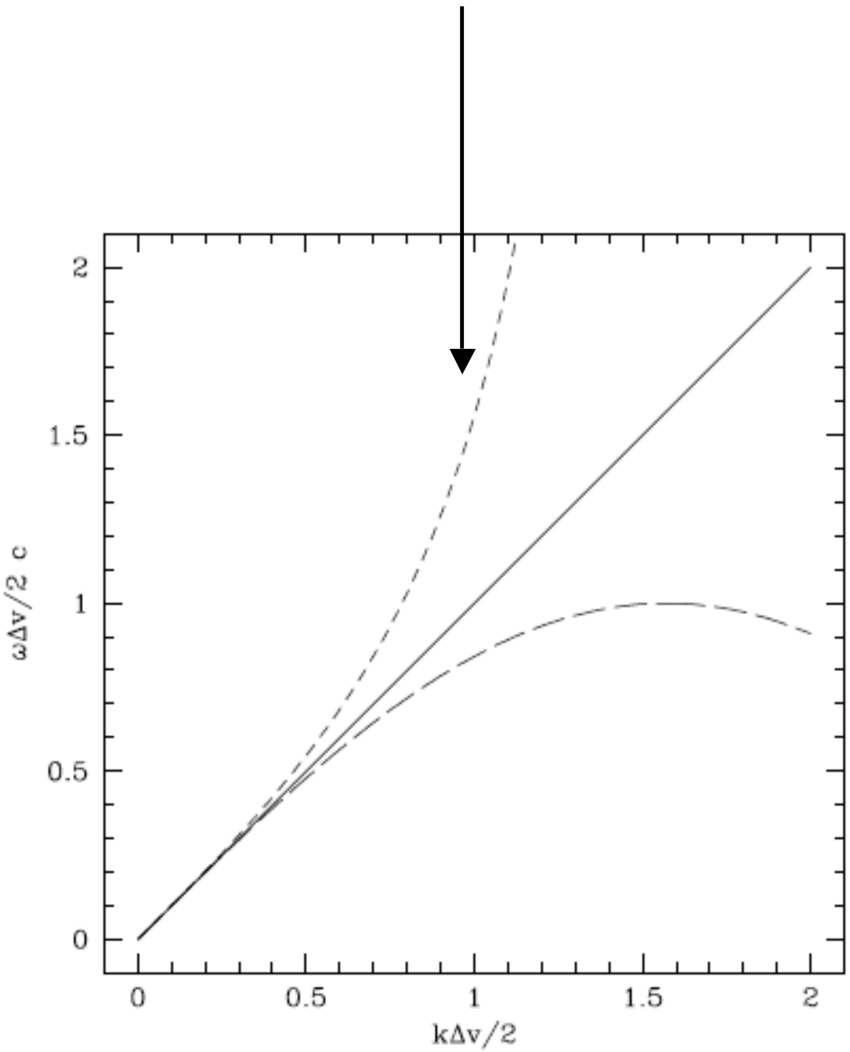
$$M_v f_j \equiv \frac{f_{j+1/2} + f_{j-1/2}}{2}$$

Formally equivalent to:

$$\partial_\phi^2 \Psi_j = \frac{3}{2}\kappa \left(1 - \frac{\Delta v^2}{4} D_v^2\right) |v| D_v (|v| D_v \Psi_j)$$

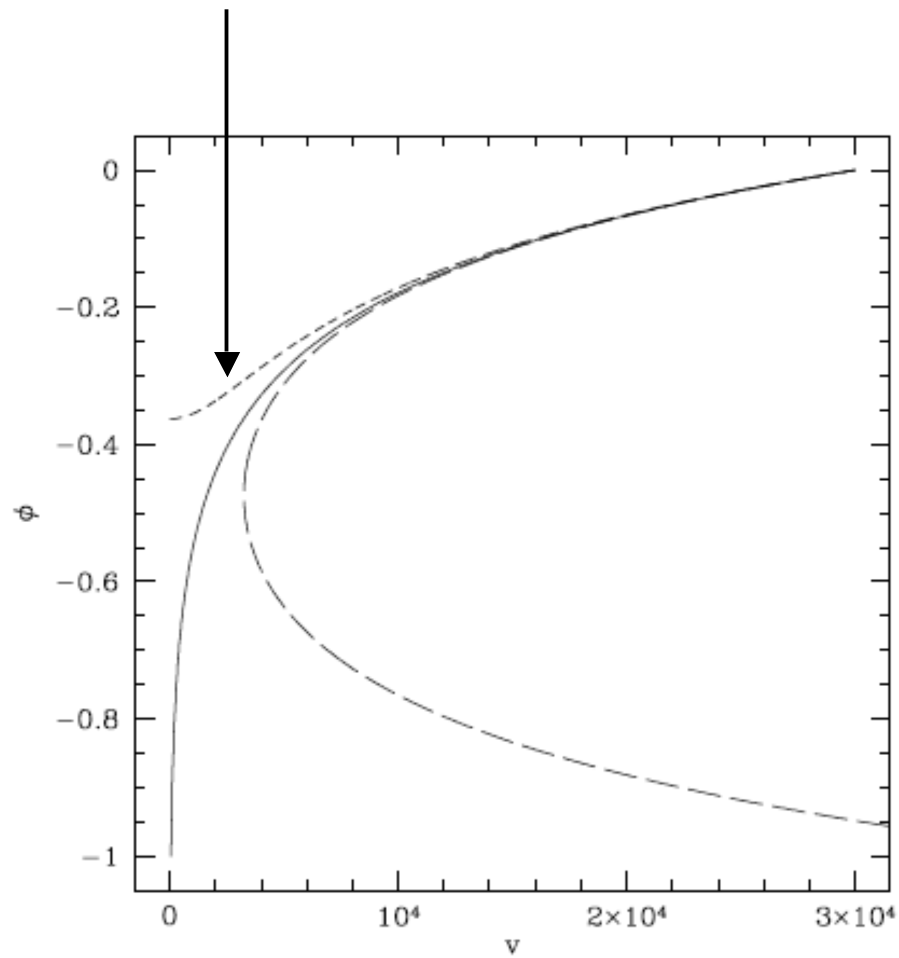
Dispersion Relation

$$\frac{\Delta v\, \omega}{2\, c} = \pm \tan\left(\frac{\Delta v\, k}{2}\right)$$

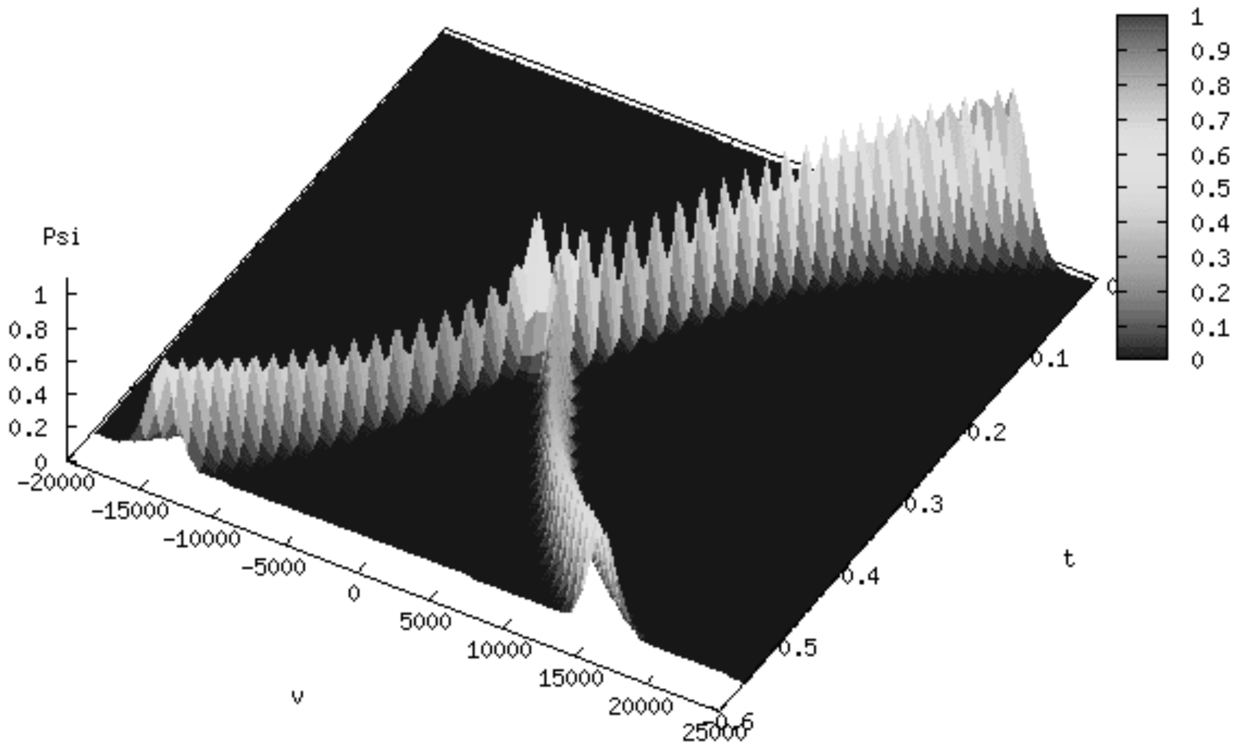


Characteristics & Group Velocity

$$V_g = \pm c \sec^2\left(\frac{\Delta v\, k}{2}\right) = \pm c\left[1 + \left(\frac{\Delta v\, \omega}{2\, c}\right)^2\right]$$



Through the Shallow Waters



Conclusions

- The Big Quantum Bounce in homogenous scalar field models is directly connected to the explicit time update in the LQC finite difference equation.
- Implicit updates or higher-order finite difference terms in more complicated models could yield evolutions that avoid the bounce.