Generalizing the Kodama State

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Introduction

- Kodama State is one of the only known solutions to all constraints with well defined semi-classical interpretation

- Represents quantum (anti)de-Sitter space

- Cosmological data suggest we are in increasingly lambda dominated universe
  - World appears to be asymptotically de-Sitter

- Exact form in connection and spin network basis: $\langle A|\Psi\rangle = N e^{\frac{3}{4k\Lambda}\int Y_{CS}[A]}$ $\langle \Gamma|\Psi\rangle = K_{\Gamma}$

But…

- Not normalizable under kinematical inner product
  - Not known to be under physical inner product
  - Linearized solutions not normalizable under linearized inner product
- Not invariant under CPT
  - Violates Lorentz invariance?
  - CPT inverted states have negative energy?
- Not invariant under large gauge transformation
  - (This could be a good thing)
- Loop transform in complex variables not as rigorous as with real variables
- Reality constraint must be implemented

Resolution

- Problems can be tracked down to complexification
    - Immirzi is pure imaginary: $\beta = \pm i$
- Need to extend state to real values of Immirzi parameter
- This can be done
- Answer is surprising:
    - Opens up a large Hilbert space of states
    - de-Sitter/Chern-Simons is one element of this class

The Generalized States

•States are WKB states corresponding to first order perturbations about de-Sitter spacetime

$$S_0 \simeq \frac{3}{2k\Lambda}\int_M \star R \wedge R - \frac{2}{\beta} R \wedge R$$

•Write action as functional of Ashtekar-Barbero connection, $A = \Gamma - \beta K$

$$S_0[A] \simeq \frac{3}{4k\Lambda\beta^3}\int_{\partial M = \Sigma} Y_{CS}[A] - (1+\beta^2)(Y_{CS}[\Gamma] - 2\beta\, K \wedge R_\Gamma)$$

•States are pure phase WKB states when Immirzi is real

$$\Psi_R[A] = N e^{iS_0[A]}$$

•States appear to depend on position *and* momentum

- •This requires careful interpretation

Interpretation

- Generalized Kodama state is like NR momentum state

$$\alpha = \frac{3(1+\beta^2)}{2k\Lambda\beta^3}$$

$$\varepsilon = \frac{3}{4k\Lambda\beta^3}$$

$$\langle x|p\rangle = \Psi_p[x] = \mathcal{N} exp[i\, x \cdot p - i\, E\, t]$$

$$\langle A|R_\Gamma\rangle = \Psi_{R_\Gamma}[A] = \mathcal{P} exp\left[i\, \alpha \int A \wedge R_\Gamma - i\, \varepsilon\, Y_{CS}[A]\right]$$

- Infinite set of states *parameterized* by curvature
- Expect different states to be orthogonal

$$\langle p'|p\rangle \sim \int dx\, e^{-ix\cdot(p'-p)} = \delta(p'-p)$$

$$\langle R'|R\rangle \sim \int \mathcal{D}\phi \mathcal{D}A\, e^{-i\alpha \int A\wedge(\phi R' - R)} = \delta(\mathcal{R}' - \mathcal{R})$$

Levi-Civita Curvature Operator

- Momentum operator in momentum basis:

$$\hat{p} = \int dp \;\; p|\Psi_p\rangle\langle\Psi_p| \;\rightarrow\; \hat{p}|\Psi_p\rangle = p|\Psi_p\rangle$$

- Similarly can construct gauge *covariant* curvature operator

$$\int_\Sigma \lambda \wedge \hat{R}_\Gamma = \int \mathcal{D}\phi \mathcal{D}\Gamma' \left[\left(\int_\Sigma \lambda \wedge \phi R'_{\Gamma'} \right) |\Psi_{\phi R'}\rangle \langle \Psi_{\phi R'}| \right]$$

$$\int_\Sigma \lambda \wedge \hat{R}_\Gamma \,|\Psi_R\rangle = \int_\Sigma \lambda \wedge R_\Gamma \,|\Psi_R\rangle$$

- Using this definition of curvature operator:

$$\hat{H}|\Psi_R\rangle = 0 \quad !!!$$

Properties

	Delta-function normalizable
	Invariance under large gauge t-forms
	Solve Hamiltonian constraint
	Semi-classical interpretation (WKB)
	CPT invariance
??	Negative Energies

References

- gr-qc/0504010, "A Generalization of the Kodama State for Arbitrary Values of the Immirzi Parameter"

- Upcoming papers:
  - Generalizing the Kodama State I: Construction
  - Generalizing the Kodama State II: Properties and physical interpretation