Singularities, Effective Actions and String Theory

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References: a biased collection

Singularities $\longrightarrow$ important role in the evolution of our still incomplete understanding of both nongravitational and gravitational theories

w/o gravity $\longrightarrow$ IR singularities: definition of physical states

quantum origin — IR-safe observables

UV singularities: high energy modes renormalized away – not always possible

What theories have a UV completion?

When is it needed? Is it unique?

– An effective theory does not know when and why it breaks down

w/ gravity $\longrightarrow$ classically cannot be avoided (Hawking, Penrose)

classical level — geodesics terminate in regions with large curvature

Should they be removed? How? Is it unique?

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Sometimes, but not always (Horowitz, Myers)

– There exist classical "geometries" which we classify as unphysical
e.g. negative-mass Schwarzschild

– If resolved and new physics is localized near the singularity
$\longrightarrow$ negative energy, unstable flat space

◇ need singularity to rule solution as unphysical

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Possible cures:	quantization	LQG & string theory
	high deriv. terms (classical)	string theory

- In string theory some GR-singular spaces are smooth...

◇ some singularities are morally-similar to QFT IR singularities – related to changes in the field content:

– away from singularities: GR is fine

– near resolved singularity: GR extended with additional light states

- corresponding fields: massive away from singularities or stuck there

Precise details depend on the type of singularity –

- orbifolds $\mathbb{C}^n/\Gamma$ (twisted fields) (Dixon, Harvey, Vafa, Witten 1985/86)
- some singular plane waves (higher derivarives?) (Horowitz, Steif 1990)
- some $\mathcal{N} = 2$ effective act. (wrapped branes) (Strominger 1995; +Vafa 1996)

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Not all cases treatable geometrically

- (extremal) black holes=ensemble of nonsingular microstates (Mathur)
- string-scale $AdS_5 \times S^5/\Gamma$; instability cured "holographically" (Dymarsky, Klebanov, RR;+Franco)

Not all singularities are smoothed out

◇ some plane waves remain singular (strings get ripped apart as they pass through the front wave) (Horowitz, Steif; David; RR)

◇ unclear how to treat most space-like singularities; sometimes some fields see a smooth geometry (Liu, Moore, Seiberg; Horowitz, Polchinski) (Buchel, Langfelder, Walcher/Dijkgraaf, Verlinde, Verlinde)

◇ orbifolds of flat space $\mathcal{M} = \mathbb{R}^{(9-2n,1)} \times \mathbb{C}^n/\Gamma$: exact CFTs

- asymptotic states propagate on $\mathcal{M}$; GR$+B_{\mu\nu} + \phi + F_{2n(+1)}$
- modular invariance $\rightarrow$ more states: asymptotic on $\mathbb{R}^{(9-2n,1)}$; localized on $\mathbb{C}^n/\Gamma$
- If no tachyons, graviton scattering amplitudes on $\mathcal{M}$ are finite.

◇ Partial resolution: analytical continuation of 2d bh $SL(2)/U(1)$

(Buchel, Langfelder, Walcher)

	momentum	winding
classical	$ds^2 = \lvert k\rvert \left(-dt^2 + \tanh^2 t \; dr^2\right)$	$ds^2 = \lvert k\rvert \left(-dt^2 + \coth^2 t \; dr^2\right)$
high der.	$ds^2 = \lvert k\rvert \left(-dt^2 + \frac{\tanh^2 t}{1+\frac{2}{\lvert k\rvert}\tanh^2 t} dr^2\right)$	$ds^2 = \lvert k\rvert \left(-dt^2 + \frac{\coth^2 t}{1+\frac{2}{\lvert k\rvert}\coth^2 t} dr^2\right)$

- due to higher derivative terms, suppressed by $1/k$: resummation

$\diamond$ $\mathcal{N} = 2$ effective actions from CY compactifications (Strominger)

- for fixed topology, the moduli space of CY manifolds contain singularities; generic type – conifold

$$u^2 + v^2 + w^2 + t^2 = 0 \quad \longrightarrow \quad u^2 + v^2 + w^2 + t^2 = \epsilon^2 \quad \text{——} \quad S^3$$

– conifold singularity appears as log singularity moduli space metric

– Moduli appear as fields in effective action; at generic point on moduli space kinetic term is given by moduli space metric

The cure:

- 3-branes wrap S^3 and are massless at singularity; effective action derived at generic point does not apply near this point
- extra field in effective action; quanta=extremal black holes!
- recover log singularity by integrating out this field

◇ less-than-trustworthy geometry

- $\mathbb{C}^3/\Gamma$ is a real cone over S^5/Γ $\rightarrow$ if localized tachyons on $\mathbb{C}^3/\Gamma$ and Γ acts w/o fixed points on S^5, then for sufficiently small S^5 S^5/Γ will still have tachyon(s)

– How comes? – twisted states get lighter as size of S^5 is decreased

  - supergravity not valid (and not trustworthy anyway $R^2_{AdS} \sim \alpha'$)
  - hard to guess how to correctly include all relevant terms
  - hard to describe the condensation of the tachyon(s)

- A solution: use holographic description

  - dual theory is *perturbative* Γ-orbifold of $\mathcal{N} = 4$ SYM
  - tachyon(s)=quantum-induced (2-trace) coupling *naively* running to negative infinity in finite RG time
  - tachyon condensation=$\langle \mathcal{O} \rangle \neq 0$ and $\Gamma(\mathcal{O}) \neq 0$

- Less naive: coupled RG flow: 2-trace+YM coupling+masses

  $M \propto \langle \mathcal{O} \rangle$

  $M \geq \Lambda$ the corresponding field decouples and RG flow changes

- Spectrum of $\mathcal{N} = 4$ SYM appears for sufficiently large $\langle \mathcal{O} \rangle$

  - rank of gauge group is reduced

  - if we are to attempt a geometric interpretation –

the condensation of the marginally-tachyonic fields on string-size $AdS_5 \times S^5/\Gamma$ leads to $AdS_5 \times S^5$ of size reduced by $|\Gamma|$

◇ holography (AdS/CFT) has interesting suggestions about the fate of black hole singularities $\longrightarrow$ Mathur's conjecture

- non-extremal black holes $\leftrightarrow$ gauge theory thermal states

$$\rho_{\text{bh}} = \sum_{E} e^{-\beta E} \, |E\rangle\langle E|$$

- entropy of extremal, asymptotically AdS_3 black holes can be computed (in controllable situations) as

$$S = k_B \ln N_{\text{op}}$$

N_{op} =nr. of operators with the same charges as the black hole

- the gauge theory deformed by a gauge-invariant operator is dual to a *nonsingular* asymptotically AdS_3 geometry; differences are "localized" near the origin

Is it possible that a(n extremal) black hole is a statistical ensemble of *nonsingular* geometrical microstates, each of them smooth and without a horizon?

Mathur's conjecture: Yes.

- horizon=where microstates "differ significantly" from each other

- The Plan: – construct the microstates and understand entropy

  – works great for 1-charge bh (Mathur, Lunin, Saxena, Maldacena, Maoz, etc)

  - entropy vanishes classically; string scale horizon; $S \propto A$

  – complicated for 2-charge bh; only few examples are known

  – arguments at linearized level

  - classically, solutions depend on continuous parameters; need to understand quantization of the space of solutions

Main point: physics "explaining" bh singularity is not localized at the classical singularity, but extends all the way to horizon

Summary

- Singularities are useful for ruling out unphysical effects
- Singularities can be artifacts of effective actions breaking down
  - signaling the appearance of additional light states
  - signaling that low energy approximation breaks down
- Geometrically, signal that classical geometry is not trustworthy
  - holographic description may be better
  - "statistical" description in terms of microstates