Effective Dynamics of Loop Quantum Cosmology

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Overview

Dynamics in loop quantum cosmology from quantum theory, difference equation [talks by Singh, Pawlowski]

Massless scalar field, k=0 cosmology, bounce predicted in backward evolution [Ashtekar, Singh, Pawlowski (2006)]

Can results be understood in "effective" classical picture which captures important physics of quantum theory?

Example: quantum mechanics

- Dynamics determined from Schrodinger equation
- Yet, in everyday regime, can describe dynamics in terms of Newtonian physics

Philosophy of effective theory of LQC:

- Quantum dynamics approximated by effective continuous equations of motion
- In semi-classical regime, the effective theory agrees with classical GR
- There is an intermediate regime where the effective theory can diverge from classical GR

Such an effective theory exists in LQC and can account for the quantum bounce to good precision!

Deriving Effective Equations of LQC

Classical framework of LQC:

Dynamical gravitational variables: connection component c, triad component p

In terms of scale factor, $p \propto a^2$

c, p canonically conjugate: $\{c, p\} = \kappa\gamma/3$

Dynamics governed by Hamiltonian constraint $\mathcal{C} = -\frac{3}{\gamma^2 \kappa}\sqrt{p}\, c^2 + p^{3/2}\rho_M$

Classical equations of motion:

Hamiltonian $\mathcal{H} = N\mathcal{C}$

Hamilton's equations: for instance $\dot{p} = \{p, \mathcal{H}\} = -\frac{\kappa\gamma}{3}\frac{\partial \mathcal{H}}{\partial c}$

Hamiltonian constraint must vanish: $\mathcal{C} = 0$

Vanishing of Hamiltonian constraint gives Friedmann equation $H^2 = (\kappa/3)\,\rho_M$

Quantization:

Constraint promoted to operator $\widehat{C}$ a la Thiemann using holonomies of connection

Physical wavefunctions annihilated by constraint operator - difference equation

Important ambiguity: length $\overline{\mu}$ of holonomies used to define constraint

Consider "improved" quantization: $\overline{\mu} \sim 1/\sqrt{p}$ decreases for larger p

Deriving Effective Equations of LQC

Effective picture of LQC:

- Several approaches to derive effective equations
- Most lead to a modified effective Hamiltonian constraint $\mathcal{C}_{eff}$
- From the effective constraint, effective equations of motion derived $\Rightarrow$ effective Friedmann equation

Example of systematic approach [Ashtekar, Bojowald, Taveras, Willis]:

- Application of geometric formulation of quantum mechanics
- Quantum Hilbert space treated as infinite dimensional phase space with structure of fiber bundle
- Classical phase (c, p) space is base space
- Vertical fibers = states with same expectation values of $< c >, < p >$
- Search for horizontal cross section using coherent states preserved by Hamiltonian flow
- Expectation value of $\widehat{C}$ on horizontal section defines $\mathcal{C}_{eff}$

Deriving Effective Equations of LQC

Geometrical formulation can be successfully applied to LQC

Classical constraint: $\mathcal{C} = -\frac{3}{\gamma^2 \kappa}\sqrt{p}\, c^2 + p^{3/2}\rho_M$

Leading order effective constraint [Taveras]:

$$\boxed{\mathcal{C}_{eff} = -\frac{3}{\kappa\gamma^2\overline{\mu}^2}\sqrt{p}\,\sin^2(\overline{\mu}c) + p^{3/2}\rho_{M\,\mathrm{eff}}}$$

Corrections appear on gravitational part and matter part (inverse p modifications appear in $\rho_{M\,\mathrm{eff}}$)

Same effective constraint can be derived through WKB techniques [Banerjee, Date, Hossain], and path integral methods [KV]

k = 0 Model

What modifications to cosmological dynamics predicted by effective theory for $k = 0$ model?

$\mathcal{C}_{eff} = -\frac{3}{\kappa\gamma^2\overline{\mu}^2}\sqrt{p}\,\sin^2(\overline{\mu}c) + p^{3/2}\rho_M$

Derive the effective Friedmann equation:

Note that $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{4}\left(\frac{\dot{p}}{p}\right)^2$ (using $p \propto a^2$)

Hamilton's equation: $\dot{p}^2 = (\frac{\kappa\gamma}{3}\frac{\partial\mathcal{C}_{eff}}{\partial c})^2 = \frac{4p}{\gamma^2\overline{\mu}^2}\sin^2(\overline{\mu}c)\,\cos^2(\overline{\mu}c)$

Vanishing of effective constraint: $\mathcal{C}_{eff} = 0 \Rightarrow \sin^2(\overline{\mu}c) = \frac{\kappa\gamma^2\overline{\mu}^2 p}{3}\rho_M$

Putting these three together gives effective Friedmann equation $\boxed{H^2 = \frac{\kappa}{3}\rho_M\left[1 - \frac{\rho_M}{\rho_c}\right]}$

Key features:

$\rho_c = \frac{3}{\kappa\gamma^2\overline{\mu}^2 p}$ is on order of Planck density in "improved" quantization of LQC ($\overline{\mu}^2 \propto 1/p$)

Turn-around ($H = 0, \dot{a} = 0$) when $\rho_M = \rho_c$

Define effective Newton's constant: $G' = G\left[1 - \frac{\rho_M}{\rho_c}\right]$ $\boxed{\text{Gravity repulsive for } \rho_M > \rho_c}$

Qualitative agreement with running G from renormalization group analysis of quantum gravity [talk by Saueressig]

k = 0 Model with Massless Scalar Field

Phenomenology of flat $k = 0$ model:

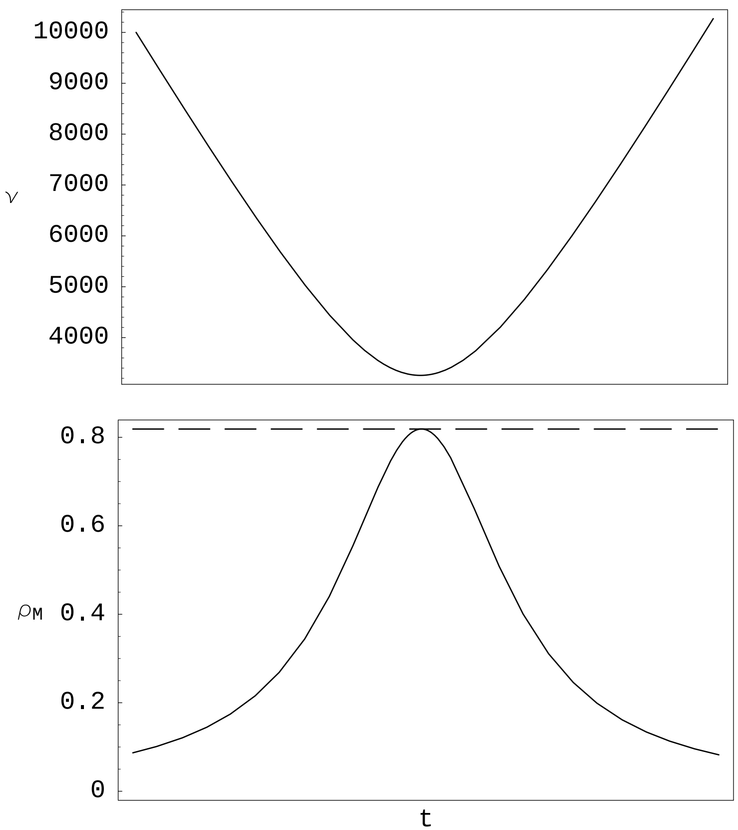
$$H^2 = \frac{\kappa}{3}\rho_M \left[1 - \frac{\rho_M}{\rho_c}\right]$$

Example: massless scalar field

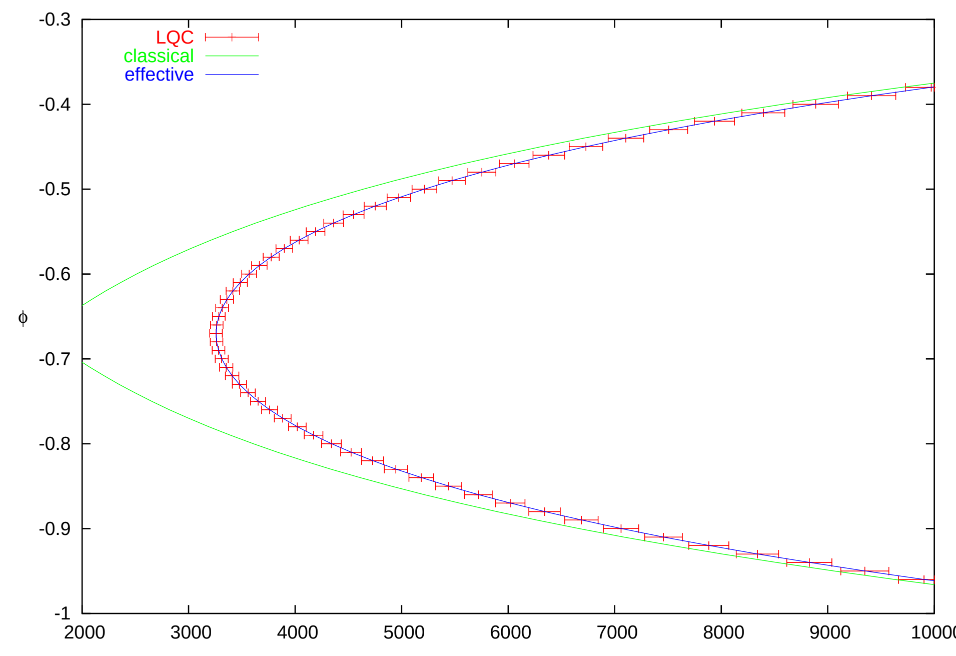
$\rho_M = \frac{p_\phi^2}{2p^3}$ (scalar field momentum p_ϕ is constant of motion)

Starting at large p, $\rho_M \ll \rho_c$ and Friedmann equation implies classical behavior $H^2 = (\kappa/3)\rho_M$

Tracing backwards in time, ρ_M increases as universe contracts, bounce when $\rho_M \sim \rho_c$



[Ashtekar, Singh, Pawlowski (2006)]



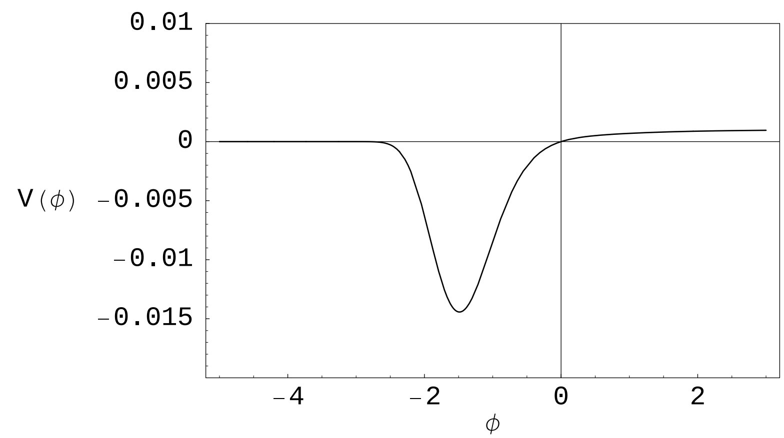
Non-Singular Cosmology

What are ramifications for more general forms of matter in flat $k = 0$ model?

Non-singular cosmologies without need for exotic matter violating energy conditions [Singh, KV, Vereshchagin (2006)]

Application: Cyclic Universe

- Visible universe confined to one of two four dimensional branes that can collide
- Brane separation described by a scalar field ϕ with potential $V(\phi)$ that is strictly negative for $\phi < 0$
- Brane collision ($\phi \to -\infty$) causes collapse of visible universe when $V(\phi) < 0$. Singularity reached at $\phi = -\infty$.
- Postulated that branes collision not singular, ϕ bounces back to $V > 0$ region, visible universe expands, cycle begins anew



Problems with Cyclic model:

- Brane collision singular in proposed models
- Matter energy density can reach super Planckian values
- Scale invariant perturbation spectrum claimed to pass through singular transition

Can LQC alleviate these problems?

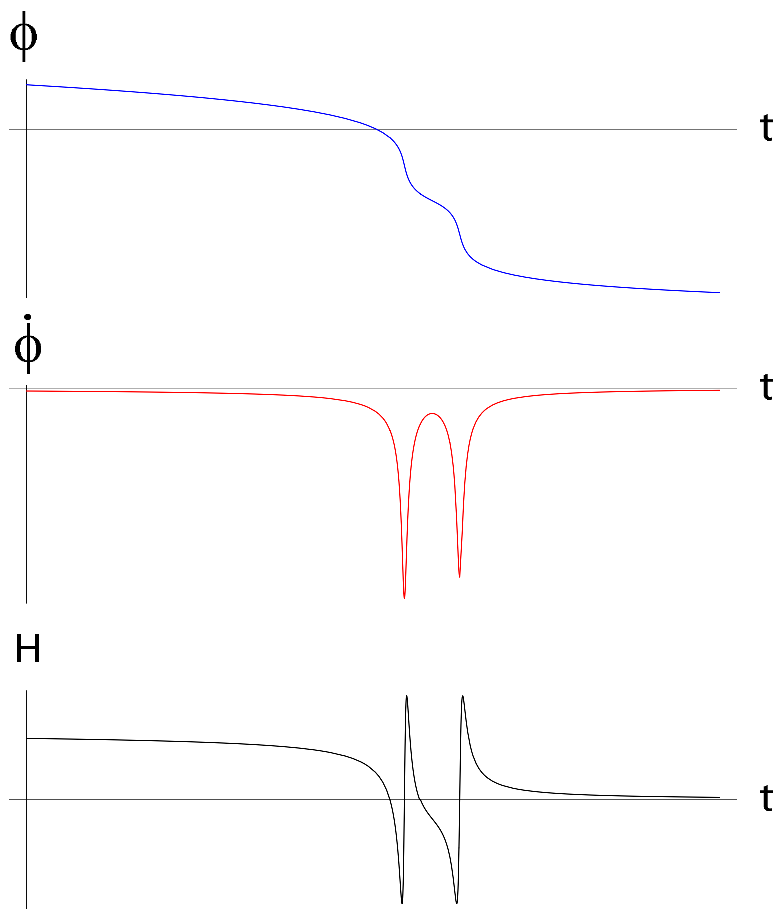
Non-Singular Cosmology

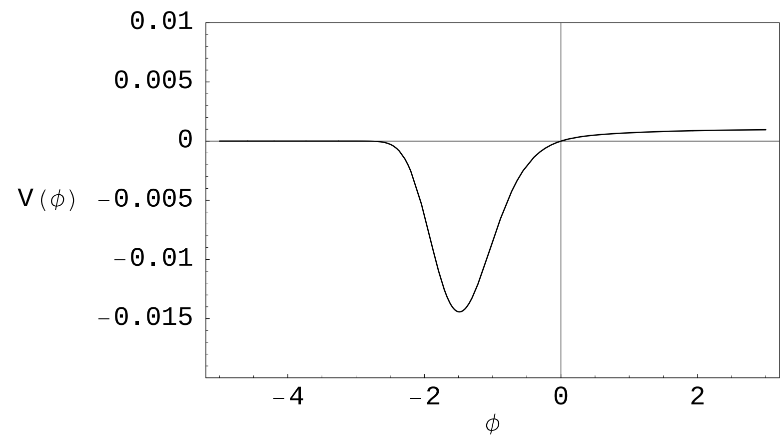
Model: Gravity on 4D visible brane includes LQC contributions

LQC effects resolve scale factor singularity and bound energy density

Scalar field does not bounce back from negative region $\Rightarrow$ non-cyclic behavior

Cyclic behavior can result if potential becomes positive in $\phi < 0$ region (perhaps from non-perturbative string effects)





k = +1 Model

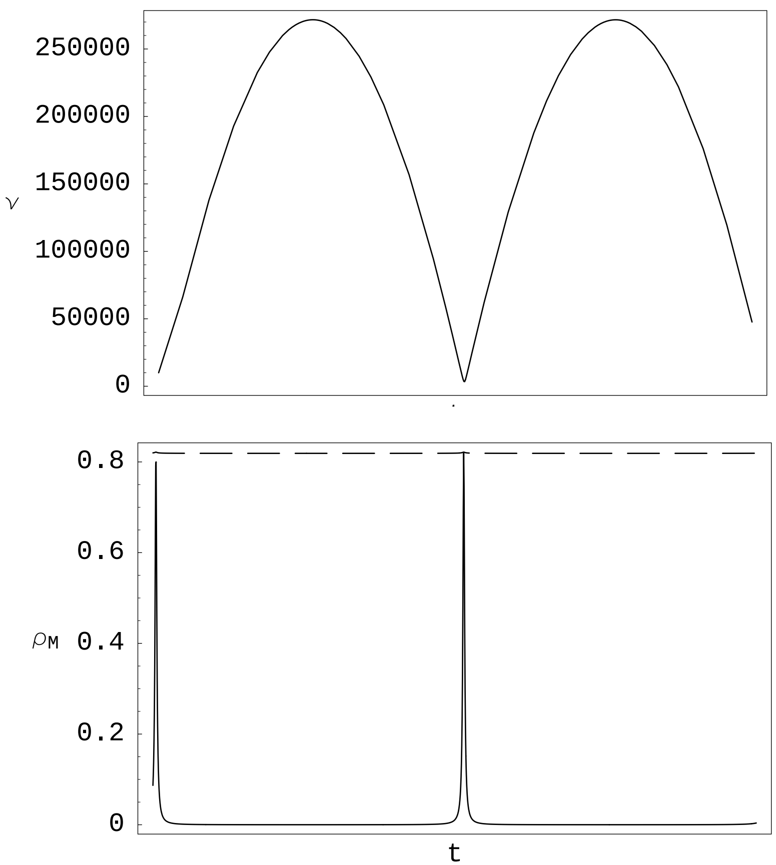
Improved quantization scheme extended to closed $k = +1$ model [Ashtekar, Lewandowski, Szulc, KV, in progress]

Classical Friedmann equation: $H^2 = \frac{\kappa}{3}\rho_M - \frac{1}{a^2}$

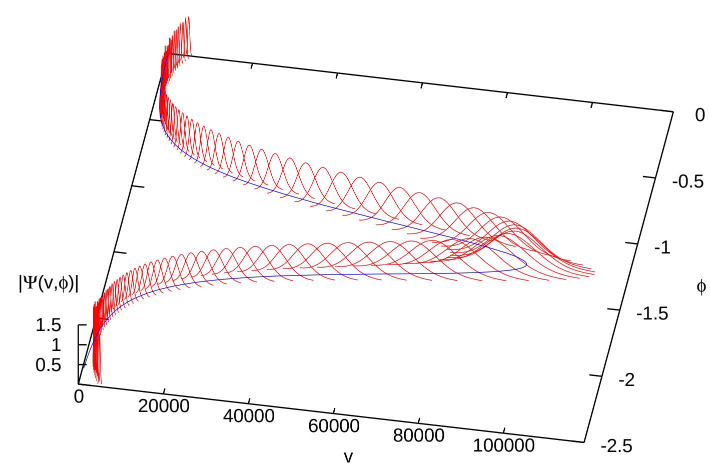
Effective Friedmann equation: $H^2 = \left(\frac{\kappa}{3}\rho_M - \frac{1}{a^2}\right)\left(1 + \frac{\gamma^2\Delta}{a^2} - \frac{\kappa\gamma^2\Delta}{3}\rho_M\right) + \mathcal{O}(1/a^4)$

Massless scalar field: cyclic behavior

Collapsing universe bounces, expands, recollapses and repeats



Quantum Evolution: [Ashtekar, Singh, Pawlowski]:



Conclusion

Effective theory useful for describing important physical features of quantum theory

Need to establish conditions of its validity (where does effective picture break down)

Effective theory can be tested on anisotropic models, and quantization of Schwarzschild singularity

For phenomenology, need to include perturbations to realistically talk about bouncing universes (work in progress - talk by Kagan)

Ultimately, extraction of physics from more realistic models (for instance with perturbations) will probably require studying the effective equations