$\boxed{\text{Loopy QM is equivalent to Schrödinger QM}}$

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- Motivation: Loopy QM models LQG

  (✓) Kinematics: Rep. of Heisenberg-Weyl alg.

  (?) dynamics: Not even the SHO Hamiltonian can be rep. in $\mathcal{H}_{poly}$

- Strategy: LQ kinematics + dynamics as continuum lim. of eff. dynamics

implemented in loopy QM

- Results: $\mathcal{H}_{phys}^{*} = \overleftarrow{\mathcal{H}}_{\mathbb{R}}^{*\,ren} \big/ \mathrm{Ker}(\,,\,)_{\mathbb{R}} \simeq L^2(\mathbb{R}, dx)$

where $\overleftarrow{\mathcal{H}}_{\mathbb{R}}^{*\,ren} = \overleftarrow{\lim}_{C_n \to \mathbb{R}} \mathcal{H}_{C_n}^{*\,ren}$

$$\begin{array}{c} \overleftarrow{\mathcal{H}}_{\mathbb{R}}^{*\,ren} \\ \cap \\ Cyl_x^{*} \end{array}$$

$H_{\mathbb{R}}: \mathcal{H}_{phys} \to \mathcal{H}_{phys}$ equiv. to H_{SHO}

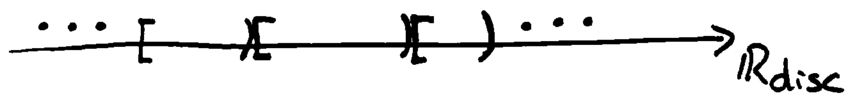
$\updownarrow$

$\{H_{C_m}^{ren}: \mathcal{H}_{C_m} \to \mathcal{H}_{C_m}\}_{d^*\text{-compat.}}$, $H_{C_m}^{ren} = \lim_{C_n \to \mathbb{R}} d^* H_{C_n}$

• Polymer representation space: $\mathcal{H}_{poly,x} \ni \delta_{x_0}(x)$ Kroneker

$\{\delta_{x_0}\}_{x_0 \in \mathbb{R}_{disc}}$ orthonormal basis

• Scale C



$$\mathcal{H}_c = \mathcal{H}_{poly,x} / \sim_c$$

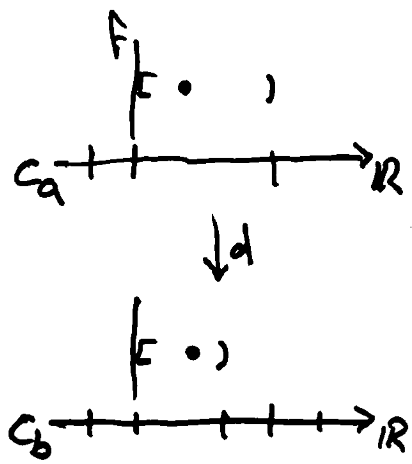
$\{e_\alpha = [\delta_{x_0}]_c\}_{\alpha \in C}$ orthonormal basis

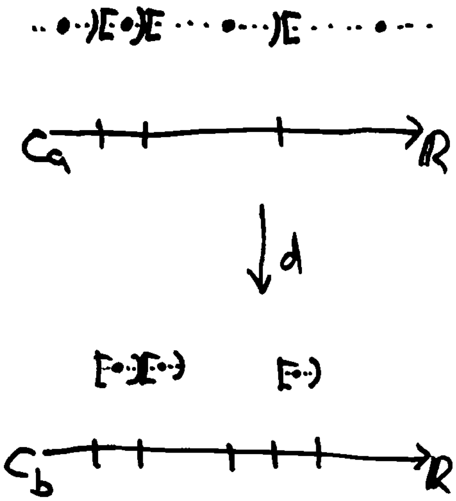
$\mathcal{H}_c^*$ is the $\sim_c$ preserving subspace of Cyl_x^*

$\{\omega_\alpha = \chi_\alpha\}_{\alpha \in C}$ dual basis

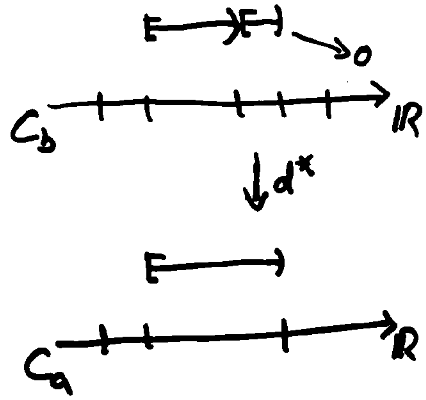
- Coarse graining $\qquad C_a \leq C_b \qquad$ partially ord. fam. of scales

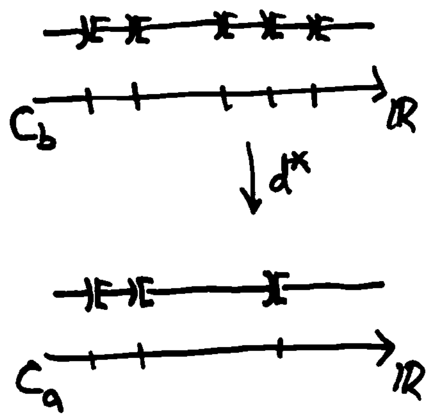
$$\mathcal{H}_{C_a} \xrightarrow{d} \mathcal{H}_{C_b}$$ decimation





$$\mathcal{H}^*_{C_a} \xleftarrow{\;d^*\;} \mathcal{H}^*_{C_b}$$





- Continuum limit (Kinematical)

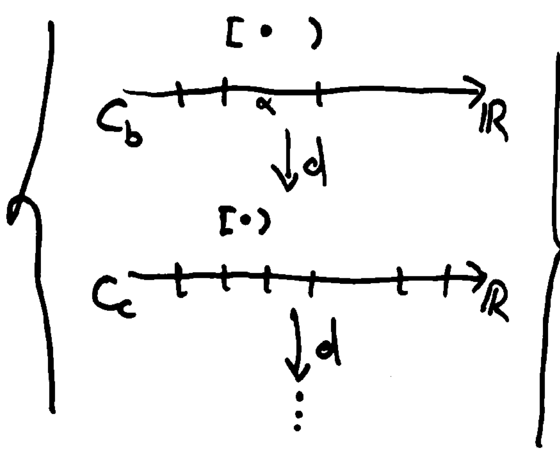
Properties of set of scales:

- Partial order $(\leq)$
- Common refinement
  $\forall\, C_a, C_b \quad \exists\, C_c : \quad C_a \leq C_c,\ C_b \leq C_c$
- Infinite refinement
  $\forall\, U \subset \mathbb{R}$ open $\exists\, C,\ \alpha \in C : \alpha \subset U$

Needed for $\lim_{C \to \mathbb{R}}$

$$\overrightarrow{\mathcal{H}}_{\mathbb{R}} := \overrightarrow{d\text{-}\lim_{C \to \mathbb{R}}}\ \mathcal{H}_C$$

$\ni$



$=: e_{L(\alpha)} \longleftrightarrow S_{L(\alpha)} \in \mathcal{H}_{poly,x}$

$$\overrightarrow{\mathcal{H}}_{\mathbb{R}} \simeq \mathcal{H}_{poly,x}$$

$$\overleftarrow{\mathcal{H}}^*_{\mathbb{R}} := \overleftarrow{d^*\text{-}\lim}_{c \to \mathbb{R}} \mathcal{H}^*_c$$

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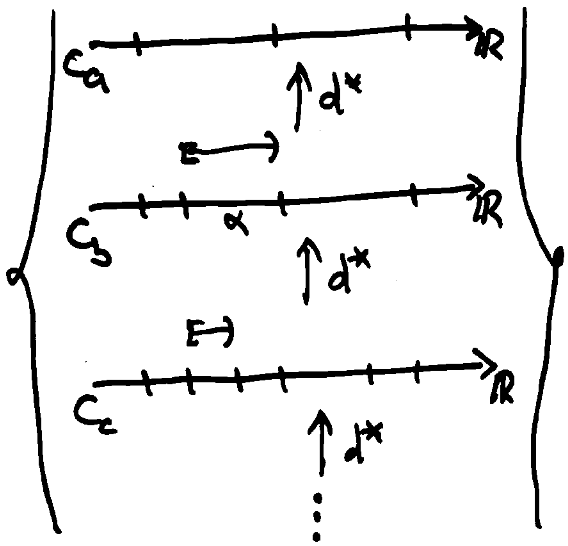


$=: W_{L(\alpha)}$

$\updownarrow$

$(S_{L(\alpha)}, -)_{poly} \in \mathcal{H}^*_{poly,x}$

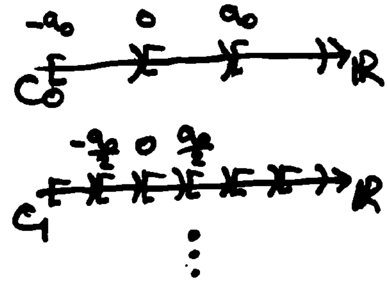
$$\overleftarrow{\mathcal{H}}^*_{\mathbb{R}} \simeq \mathcal{H}^*_{poly,x} \subset Cyl|^*_x$$

Kinematics of Loopy QM

- Dynamics as c. lim of effective dynamics

Regularization on regular decomps.

Renorm. of Hamiltonian



$$C_0 \qquad C_1 \qquad C_2 \qquad \cdots \qquad \mathbb{R}$$

$$H_{c_0}$$

$$H_{c_0(c_1)} \xleftarrow{d^*} H_{c_1}$$

$$H_{c_0(c_2)} \xleftarrow{d^*} H_{c_1(c_2)} \xleftarrow{d^*} H_{c_2}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$H^{ren}_{c_0} \xleftarrow{d^*} H^{ren}_{c_1} \xleftarrow{d^*} H^{ren}_{c_2} \xleftarrow{d^*} \cdots \xleftarrow{d^*} \{H^{ren}_{c_m}\}_{d^* comp}$$

SHO

$$H_{c_m} : \mathcal{H}_{c_m} \to \mathcal{H}_{c_m}$$ borrowed from Ashtekar, Fairhurst, Willis $$H_{\Gamma_m} : \mathcal{H}_{\Gamma_m} \to \mathcal{H}_{\Gamma_m}$$

- Convergence of $H^{ren}_{c_m} := \lim\limits_{c_n \to \mathbb{R}} d^* H_{c_n}$

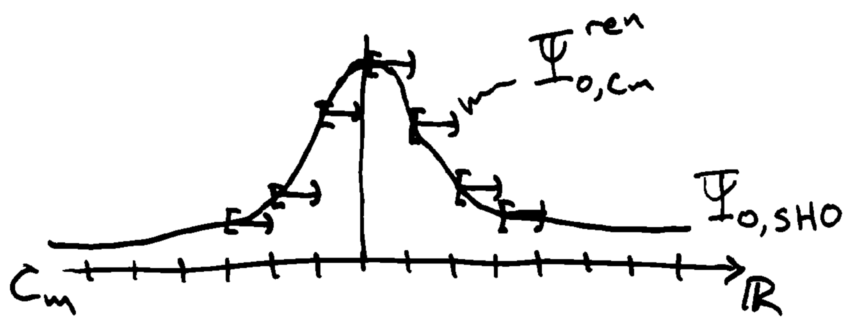
$$H_{c_m} = \sum_{v=0}^{v_{max}} E_{v,c_m} \, \Psi_{v,c_m} \otimes \Psi_{v,c_m} + H_{cont.}$$

where $\Psi_{v,c_m} \in \mathcal{H}^*_{c_m} \subset C_Y|^*_x$

$$\lim_{c_n \to \mathbb{R}} E_{v,c_n} = E_{v,SHO}$$

$$\lim_{c_n \to \mathbb{R}} d^* \Psi_{v,c_n} = \Psi^{ren}_{v,c_m} \in \mathcal{H}^*_{c_m} \subset C_Y|^*_x$$

using numerical result A-F-W



Moreover $\lim\limits_{c_m \to \mathbb{R}} \Psi^{ren}_{v,c_m} = \Psi_{v,SHO} \in C_Y|^*_x$

However $\{\Psi^{ren}_{\nu,c_m}\}_{d^*\text{-comp}} \notin \overleftarrow{\mathcal{H}}^{*}_{\mathbb{R}}$

$$\|\Psi^{ren}_{\nu,c_m}\|_{Kin,c_m} \longrightarrow \infty$$

- Physical Hilbert space

$\mathcal{H}^{*\,ren}_{c_m}$ means $\mathcal{H}^{*}_{c_m}, \frac{1}{2^m}(\ ,\ )_{Kin,c_m}$

$$\mathcal{H}^{*ren}_{c_0} \xleftarrow{d^*} \mathcal{H}^{*ren}_{c_1} \xleftarrow{d^*} \cdots \xleftarrow{d^*} \mathcal{H}^{*ren}_{\mathbb{R}} := \overleftarrow{d^*\text{-}\lim_{c_n\to\mathbb{R}}}\ \mathcal{H}^{*\,ren}_{c_n}$$

$$\cap$$

$$\mathrm{Cyl}^*_x$$

$$\boxed{\mathcal{H}^*_{phys} := \overleftarrow{\mathcal{H}}^{*\,ren}_{\mathbb{R}} \Big/ \mathrm{Ker}(\ ,\ )^{ren}_{\text{[redacted]}\,\mathbb{R}} \simeq L^2(\mathbb{R}, dx)}$$

- $H^{ren}_{c_m,\nu_{max}} = \sum_{\nu=0}^{\nu_{max}} E_\nu\, \Psi^{ren}_{\nu,c_m} \otimes \Psi^{ren}_{\nu,c_m}$

- Symmetries before and after renorm.

- $\mathcal{H}^*_{c_m}$ infinitely bet than $\mathcal{H}_{\Gamma_m}$