Causal Sets: Overview and Status

Luca Bombelli

University of Mississippi

QGA3 Conference, 25 Aug 2006

Introduction

Causal Sets

A causal set is a partially ordered set, meaning that $\forall\ x,\ y,\ z$

$$x \prec y\ ,\ \ y \prec z\ \ \Rightarrow x \prec z$$
$$x \prec y\ ,\ \ y \prec x\ \ \Rightarrow x = y$$

which is locally finite, meaning that $\forall\ x,\ y$

$$\text{card}\{z \mid x \prec z \prec y\} < \infty\ .$$

Causet elements are interpreted as events in a discretized spacetime, and the order relation as a causal relation.

Spacetime as a Causal Set

The idea that quantum spacetime can be modeled as a causal set, proposed by Sorkin and collaborators [Bombelli et al 1987, and precursor proposals by 't Hooft 1978, Myrheim 1978], combines:

- Spacetime metric = conformal structure + volume element (motivated by various arguments, from classical/axiomatic ones to quantization; results by Hawking et al and Malament).
- Spacetime discreteness or "atomicity" (also motivated by various arguments, from mathematical-type ones to ultraviolet cutoffs for quantum gravity).

The two ideas work particularly well together. In particular, in a discretized setting it is natural to replace "conformal structure" by "causal structure" and "volume element" by "volume".

Causal Sets and Continuum Geometries

General Considerations

When does c correspond to some (M, g)? When there is an embedding $i : c \to M$, which is "faithful" in the sense that:

- Partial order = Causal relations: $x \prec y \Leftrightarrow i(x) \in I^-(i(y))$,
- The embedded points are uniformly distributed in (M, g) (in a covariant sense; in particular, randomly distributed, which implies an effective local Lorentz invariance, even for a single causet),
- (M, g) has no length scales smaller than the embedding one.

Idea: Under these conditions c determines (M, g) "uniquely", since there is no structure at small scales and at larger ones, volumes $V(R) = |i(c) \cap R|$ and light cones are determined by $\prec$.

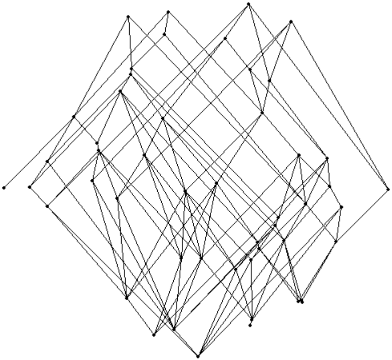

Figure: A 50-element causal set generated by a uniform random sprinkling in (a diamond-shaped region of) 2D Minkowski space.

The Central Conjecture and The Inverse Problem

- *Original version:* If a causet C can be faithfully embedded in two Lorentzian manifolds (M, g) and (M', g'), then the latter are close, up to small variations on small scales.

- *In practice:* The conjecture has proven difficult to formulate precisely, let alone prove. So, try to characterize (classes of) causets that do correspond to manifolds, and find out how to extract from those causets geometrical information.

- *Strategy:* Simulate a faithful embedding in some (M, g) by sprinkling points at random in it and inducing a partial order among them; try to read off properties of (M, g) from combinatorial ones of C; if successful, that property is uniquely determined by C within the class of geometries considered.

Progress

- *Limiting case:* In the infinite point density or $\kappa\hbar \to \infty$, the conjecture has been proved [Sorkin; Bombelli & Meyer 1989].
- *Dimension:* Two main approaches, finding subsets that force a certain dimensionality or using statistical methods. For posets uniformly embedded in Minkowski space, there are several good dimension estimators [Meyer 1988, 1993; Reid 2002].
- *Timelike distances:* For two points that are related, the length of the longest chain in the poset (geodesic) is a good estimator of the timelike separation between the corresponding manifold points [Brightwell & Gregory 1991; Ilie et al 2005].
- *In general:* There is an algorithm that will try to embed a causet in 2D Minkowski space, which in principle can be extended to any dimensions [Henson 2006].

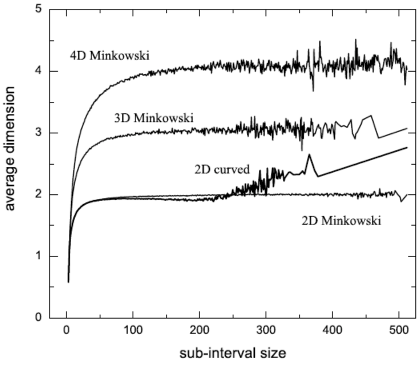

Figure: Dimension estimate calculated by the midpoint method for causets sprinkled in Minkowski space of various dimensionalities and one conformally flat 2D spacetime [Reid 2002].

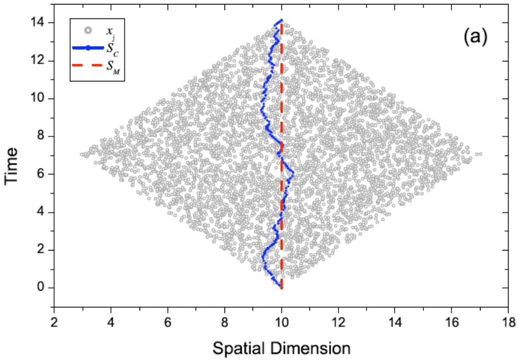

Figure: Longest path between two points in 2D Minkowski space [Ilie et al 2005].

Remarks on Kinematical Aspects

• *Spacelike hypersurfaces:* Thickened hypersurfaces, corresponding to a number of layers in a causet, can be used to define spatial topologies [Major et al 2006] (which can be expected to be 2D at small scales and higher-dimensional at larger scales).

• *Coarse-graining:* From a causet, one can obtain a coarse-grained version by choosing a number $0 < p < 1$, and retaining each element with probability p. Changes the "scale" of the causet, and may be needed for some causets that cannot be embedded in any (almost) flat manifold.

• *Kinematical flexibility:* The framework can naturally accommodate topology change (if not mediated by causality violations) and a scale-dependent effective topology, including the dimensionality of a causet, as determined by a faithful embedding.

Dynamics of Causal Sets

General Considerations

• *Approach:* In a generalized sum-over-histories formulation, the goal is to assign a suitable quantum measure to each covariantly defined set of histories.

• *First attempt:* Try to mimic R and the continuum action in terms of combinatorial quantities; Not as easy as it sounds, relies on a much better understanding of the kinematics than we one we currently have.

• *Alternative:* Instead of considering a causet like a "block universe", set up an evolution framework, dynamics as a sequential growth process (discrete!); Probably easier to interpret.

Classical Stochastic Evolution

• *Warmup:* Possible classical versions of causet dynamics of the "stochastic sequential growth" type have been studied.

• *Possibilities:* The number of possible evolution laws is huge, so narrow down the search by imposing some set of conditions:

- Either use general principles – covariance and causality,
- Or the fact that they should give manifold-like causets.

Don't we want both of these? Well, yes, but although much progress has been made, it is not clear that one can have both.

Rideout-Sorkin Models

If one imposes discrete versions of

- *General covariance:* The probability of growing a poset c is independent of the order in which elements are added, and
- *Bell causality:* Relative probabilities for transitions are not influenced by what happens in unrelated parts of the poset,

then the possible stochastic evolutions are the Rideout-Sorkin "generalized percolation" models, each characterized by non-negative real numbers t_0, t_1, t_2, ..., where t_k is related to the probability that a new "birth" in the causet has some k-element subset to its past [Rideout & Sorkin 1999].

These conditions imply that every element has a descendant, and therefore infinitely many, and one obtains a probability measure on the set of infinite, past-finite causets.

Remarks on Dynamics

- *Resulting spacetimes:* Generically, not manifold-like.

- *Other possible approach:* To understand this apparent conflict, one can try to derive "phenomenological" dynamical rules from the requirement that they must produce causets that look like manifolds, and then check the extent to which covariance and causality are satisfied.

- *Observables:* Given any covariantly defined $A \subset \Omega$, except for a set of measure zero in the Rideout-Sorkin measures, one can find out if some c is in A from the answers to a series of questions of the type: Is a causet d a stem in c? [Brightwell et al 2002]

Phenomenology

Cosmology

• *Cosmological constant:* Using the “unimodular” approach to the dynamics of gravity, in which time = spacetime volume, Λ appears as a variable conjugate to cosmic time, and its value is subject to fluctuations whose size can be estimated from the uncertainty principle. Led to the prediction that $\Lambda \approx 1/\sqrt{V}$ [Sorkin 1990].

• *Typical evolution:* In the Rideout-Sorkin models, the universe geenerically goes through an infinite series of expansion and contraction cycles, mediated by “posts”.

• *Cosmic renormalization:* Each cycle leads to a change in the effective values of the “coupling constants” t_k; after many cycles, the universe is typically very large and the t_k converge to asymptotic values [Sorkin 2000, Martin et al 2001].

Black Hole Entropy

Counting of black hole horizon states as links and black hole entropy. Gives a number proportional to the area with the same coefficient in very different models [Dou 1999; Dou & Sorkin 2003].

Matter Models

- Simple models in which a particle moves from point to point on a causal set; Leads to a predicted random acceleration or "swerving" for free particles in Minkowski [Dowker et al 2003, Kaloper & Mattingly 2006].
- Add matter on top of the causet (fields defined at points, or links, ..., internal Hilbert spaces at points, ...).
- Hope that the causet will give rise to effective matter configurations upon coarse-graining.
- Follow up on the fact that the probabilities for causets are equivalent to those of an Ising model at the relations $x \prec y$.

Summary and Conclusions

The causal set approach is not a very radical one, spacetime is real and its connection to geometry in principle clear. But it does take the point of view that it pays off to depart from step-by-step approaches and take the suggestions of local finiteness seriously. As a consequence levels of difficulty of issues are shifted:

- Some questions are answered immediately ($D+1$ signature),
- Some conceptual questions are easier to address (observables);

One also never takes a continuum limit. On the other hand,

- Direct computations are almost impossible, even for simple models (and for long numerical progress has been slow).

However, the number of people working in the field, the level of activity and number of ideas have gone up in the past few years.

Things to Do

- Come up with a precise Hauptvermutung; try to prove it.
- Even if a proof is found, it will probably not provide much constructive information, so find ways to extract geometry from causets using simulations.
- Explore further the properties of classical stochastic dynamics, and the relationship between covariance + causality and "semiclassical" causets.
- Find a quantum measure for causal set histories.
- Try to build a causal set theory for 2D quantum gravity.

Reviews

R Sorkin, `gr-qc/0309009`;
F Dowker, `gr-qc/0508109`;
J Henson, `gr-qc/0601121`